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On the treatment of NN interaction effects in meson production in NN collisions

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Abstract

We clarify under what circumstances the nucleon–nucleon final state interaction fixes the energy dependence of the total cross–section for the reaction $NN \to NNx$ close to production threshold, where x can be any meson whose interaction with the nucleon is not too strong. We strongly criticize the procedure used recently by several authors to include the final state interaction in the reactions under discussion. In addition, we give a formula that allows one to estimate the effect of the initial state interaction for the production of heavy mesons.

Keywords: final state interaction, initial state interaction, meson production

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As early as 1952 K. Watson pointed out under what circumstances one expects the final state interaction to strongly modify the energy dependence of the total production cross–section $NN \to NNx$ [1] given by

$$\sigma_{NN\to NNx}(\eta) \propto \int_0^{m_x\eta} d\rho(q')|A(E,p')|^2 \ .$$

In the above equation q' is the momentum of the outgoing meson and $A(E,p')$ is the $NN \to NNx$ transition amplitude, which, for future convenience, is

expressed as a function of the total energy E and the relative momentum of the two nucleons in the final state p' (note, that p' and q' are related to each other via energy conservation). The phase space is denoted by $d\rho(q')$ and η denotes the maximum momentum of the emitted meson in units of its mass. Watson [1] argues that if there is a strong and attractive force between two of the outgoing particles, as is the case for the reactions under consideration, the energy dependence of the total cross–section is determined by the phase space and the energy dependence of the relevant attractive interaction, i.e.,

$$\sigma_{NN\to NNx}(\eta) \propto \int_0^{m_x\eta} d\rho(q')|T(p',p')|^2 \propto \int_0^{m_x\eta} d\rho(q')\left(\frac{\sin\delta(p')}{p'}\right)^2 \ , \qquad (1)$$

In the above equation $T(p',p')$ is the on–shell NN T–matrix, $\delta(p')$ denotes the NN phase shifts at the energy, $2E(p')$, of the final NN subsystem (here restricted to s–waves), where $E(p') \equiv \frac{p'^2}{2m}$, with m denoting the nucleon mass. When data for the reaction $pp \to pp\pi^0$ close to threshold became available [2] eq. 1 indeed turned out to give the correct energy dependence of the total cross–section [3]. Several authors [4–9] concluded from this observation that it is appropriate to calculate the transition $NN \to NNx$ to lowest order in perturbation theory and just include the final state interaction (FSI) by using a formula of the type in eq. 1; they implement the FSI by use of just the on–shell NN T–matrix, not only to get the right energy dependence of the cross–section, but also to get the strength of the matrix elements. In this letter we criticize this procedure. We shall demonstrate that the observation that the energy dependence of the cross–section is given by the *on–shell* FSI does not necessarily imply that the strength of the matrix elements is also determined by the on–shell NN interaction. We also show that Watson's requirement that the FSI be attractive in order to obtain the energy dependence of the cross–section given by eq. 1 is unnecessary. Finally we give an expression that allows one to estimate the effect of the initial state interaction (ISI) on the reaction $NN \to NNx$, with x any meson heavier than the pion, in terms of the (on–shell) NN scattering phase shifts and inelasticities.

The starting point of the present investigation is the decomposition of the total transition amplitude into a production amplitude, hereafter called M, and the NN FSI (see also Fig. 1). As M is not specified, no approximation is involved in this decomposition. Schematically we can write

$$A \ = \ M \ + \ TGM \ .$$

An integration over the intermediate momenta is needed to evaluate the second term on the right hand side. This is actually the term where the off–shell information of both the NN T–matrix and M enters, as will become clear below. To be concrete, we use non–relativistic kinematics for simplicity. The

generalization to a fully relativistic treatment is straightforward and does not provide any new insights. In addition, since we only want to investigate effects of the FSI on the energy dependence of the total cross–section, overall constant factors are dropped. Using [10]

$$G(E,k) = \mathbf{P}\frac{1}{E \; - \; 2E(k)} \; - \; i \; \pi\delta(E - 2E(k)) \; ,$$

where $\mathbf{P}$ denotes the principal value, we write the total transition amplitude A in the form

$$A(E,p') \;=\; M(E,p') \left\{ 1 - i\kappa(p')T(p',p')[1 + \frac{i}{ap'}\mathcal{P}(E,p')] \right\} \; . \qquad (2)$$

where $\kappa(p') = \frac{\pi p' m}{2}$ is the phase space density; the factor of $1/ap'$, with a denoting the low-energy NN scattering length, has been introduced for future convenience. Also, for convenience, we display only those arguments of M that are relevant for the present discussion, that is the total energy E and the relative momentum p' of the two nucleons in the final state. As pointed out in ref. [1], M depends weakly on E if the production mechanism is short ranged. In the above equation, all the off–shell effects are contained in the function $\mathcal{P}(E,p')$, whose explicit form is

$$\mathcal{P}(E,p') = \frac{ap'}{\kappa(p')}\mathbf{P}\int_0^\infty dk \frac{k^2 f(E,k)}{E - 2E(k)} = \frac{2a}{\pi}\int_0^\infty dk \left[\frac{k^2 f(E,k) - p'^2}{p'^2 - k^2}\right] \; , \qquad (3)$$

with the function f defined as

$$f(E,k) \;=\; \frac{T(p',k)}{T(p',p')}\frac{M(E,k)}{M(E,p')} \;=\; \frac{K(p',k)}{K(p',p')}\frac{M(E,k)}{M(E,p')} \; . \qquad (4)$$

The last equality in the above equation follows from the half–off–shell unitarity relation of the NN T–matrix, namely

$$T(p',k) = \frac{1}{2}\left(\eta(p')e^{2i\delta(p')} + 1\right) K(p',k)$$

with the K–matrix real by definition. Therefore, all of the imaginary part of f – and therefore of $\mathcal{P}$ – is introduced by the production amplitude M. In the latter formula use has been made of the fact that the on–shell T–matrix and the phase shifts are related by

$$\kappa(p')T(p',p') \;=\; \frac{i}{2}\left(\eta(p')e^{2i\delta(p')} - 1\right) \; , \qquad (5)$$

where $\eta(p')$ denotes the inelasticity.

Substituting eq. 5 into eq. 2, we get for the transition amplitude

$$A(\,E\,,p') = \frac{1}{2}M(E,p')e^{i\delta(p')} \times\left[\left(\eta(p')e^{i\delta(p')}+e^{-i\delta(p')}\right)-\frac{1}{i}\left(\eta(p')e^{i\delta(p')}-e^{-i\delta(p')}\right)\frac{1}{ap'}\mathcal{P}(E,p')\right]\,. \quad (6)$$

For energies near the production threshold energy one has $\eta(p')=1$, so that,

$$A(E,p') \;=\; M(E,p')e^{i\delta(p')}\left[\cos(\delta(p'))-\frac{\sin(\delta(p'))}{ap'}\mathcal{P}(E,p')\right]\,. \qquad (7)$$

Using the effective range expansion

$$p'\cot(\delta(p')) = -\frac{1}{a}+\frac{1}{2}\Lambda^2\sum_{n=0}^{\infty} r_n\left(\frac{p'^2}{\Lambda^2}\right)^{n+1} \qquad (8)$$

eq. 7 can be further reduced to

$$A(E,p') = -M(E,p')e^{i\delta(p')}\left(\frac{\sin(\delta(p'))}{ap'}\right) \times\left[\;\mathcal{P}(E,p')+1-\frac{1}{2}ar_o p'^2-...\;\right]\,. \quad (9)$$

This is the central formula of the present discussion. It reveals a number of important features. First of all, it shows that the energy dependence of the total cross–section is, indeed, given by eq. 1 as has been shown by Watson, provided the production amplitude $M(E,p')$ and the function $\mathcal{P}(E,p')$ have a weak energy dependence compared to that due to the FSI. Secondly, it is not necessary that the FSI be attractive in order for the total cross section to have the energy dependence given by eq. 1: as long as $M(E,p')$ and $\mathcal{P}(E,p')$ have a weak energy dependence, the energy dependence of the total cross–section will be given by the FSI times phase space for $p'^2 \ll (ar_0)^{-1}$. Thirdly, and most relevant to the present discussion, the above formula also shows that the strength of the amplitude $A(E,p')$ depends on the function $\mathcal{P}(E,p')$. Using just the on–shell T–matrix as the FSI instead of the full half–off–shell T–matrix means setting the function $\mathcal{P}(E,p')$ to zero. As has been mentioned before, the function $\mathcal{P}(E,p')$ summarizes all the off–shell effects of the FSI and production amplitude. As such, it is an unmeasurable and model–dependent quantity. In

particular, it depends on the particular regularization scheme used. For example, in conventional calculations based on meson–exchange models, where the regularization is done by introducing form factors, the function $\mathcal{P}(E, p')$ is very large and cannot be neglected [1] . Other regularization schemes, however, may yield a vanishing function $\mathcal{P}(E, p')$. Since the total amplitude $A(E, p')$ should not depend on the particular regularization scheme, the production amplitude $M(E, p')$ in eq. 9 must depend on the regularization scheme in such a way to compensate for the regularization dependence of $\mathcal{P}(E, p')$. At this stage one has to conclude that the procedure of just evaluating M in the on–shell tree level approximation and simply multiplying it with the on–shell NN T–matrix without consistency between the NN scattering and production amplitudes (as it was done in [4–9]) is not acceptable in order to obtain *quantitative predictions*. In the appendix we develop a simple model in order to gain more insight on this issue.

All the above considerations are not restricted to the NN final states; whenever there is a strong two–particle correlation in the final state, the energy dependence of a total production cross–section is given by the on–shell phase shifts of two of the outgoing particles. This condition is for example also met in the reaction $pp \to pK\Lambda$, as demonstrated in ref. [11].

The situation is very different for the effect of the NN interaction, responsible for the initial state distortions. Since the kinetic energy of the initial state has to be large enough to produce a meson, the NN ISI is evaluated at large energies. Therefore, in this regime we expect the variation with energy of the ISI to be small. [2] At least in the case of meson–exchange models this implies a flat off–shell behavior of the NN T–matrix at a given energy, in which case the principal value integral is expected to be small, as can be seen from eq. 3. It is this observation that allows us to use eq. 6 to estimate the effect of the ISI on the total production cross–section for the production of heavier mesons. The ISI therefore leads to a reduction of the total cross–section of the order of

$$\begin{aligned}\lambda =& \left|\frac{1}{2}e^{i\delta_L(p)}\left(\eta_L(p)e^{i\delta_L(p)} + e^{-i\delta_L(p)}\right)\right|^2 \\ =& \eta_L(p)\cos^2(\delta_L(p)) + \frac{1}{4}[1-\eta_L(p)]^2 \leq \frac{1}{4}[1+\eta_L(p)]^2 \ , \end{aligned} \tag{10}$$

where p denotes the relative momentum of the two nucleons in the initial state with the total energy E. The index L indicates the quantum numbers

[1] We checked this numerically.
[2] In case of pion production the phase shifts of the $^3P_0$ partial wave, which is the initial state for the s–wave π^0 production, still vary reasonably rapidly with energy. Therefore we do not expect the principal value integral to be small.

of the corresponding initial state. Note that, for production reactions close to threshold, selection rules strongly restrict the number of allowed initial states. In the literature there is one example that quantifies the effect of the ISI for meson production reactions, namely ref. [12], where the reaction $pp \to pp\eta$ is studied. The inclusion of the ISI in this work leads to a reduction of the total cross–section by roughly a factor of 0.3. At threshold only the $L = {}^3P_0$ state contributes to the ISI. The phase shifts and inelasticities given by the model used in [12] for the ISI are $\delta_L(p) = -60.7^o$ and $\eta_L(p) = 0.57$ [13] at $T_{Lab} = 1250$ MeV. These values agree with the phase shift analysis given by the SAID program [14]. Both phase shifts and the inelasticity vary by 10% only over an energy range of 500 MeV [14]. Using the above mentioned values for $\delta_L(p)$ and $\eta_L(p)$ we get for the reduction factor λ, defined in eq. 10, a value of 0.2. Therefore, in the case of the kinematics of the ISI for the η production, the principal value integral – within the meson–exchange model used – indeed turns out to be a correction of the order of 20% compared to the leading on–shell contribution.

In summary, we have demonstrated that the way the FSI is treated in a series of recent papers is unjustified to achieve quantitative predictions. It should be clear, however, that this does not fully disregard the results of refs. [4–9]: For the purpose of investigating the relative importance of different contributions to the production process the approach used there is still justified. Our criticism is not to the result of ref. [1]. In fact, our function $\mathcal{P}(E, p')$ appearing in eq. 2 is related to the factor $(f(r), \bar{R})$ in eq. (32) of ref. [1], where $f(r)$ accounts for the short–range behavior of the strongly interacting particles in the final state. Note that Watson [1] does not give a prescription how to calculate the overlap integral $(f(r), \bar{R})$, which would be required to fix the overall normalization. We emphasize that we do *not* claim that off–shell effects are measurable [15]. The result of this paper is the demonstration of the necessity to properly account for loop effects of the FSI in situations where the latter strongly influences the energy dependence of the total cross–section as in meson production in NN collisions. In addition, we have given a compact formula that allows one to estimate the effect of the ISI in terms of the phase shifts and inelasticities of NN scattering. This formula should prove to be useful for theoretical investigations of the production of heavy mesons close to their production threshold.

Appendix

In this appendix we will use a simple model to demonstrate the need for a consistent treatment of both the NN scattering and production amplitudes in order to obtain quantitative predictions of meson–production reactions.

Let us assume a separable NN potential

$$V(p', k) = \alpha g(p') g(k) \ , \tag{11}$$

where α is the coupling constant and $g(p')$ an arbitrary real function of p'. With this potential the T–matrix scattering equation can be readily solved to yield

$$T(p',k) = \frac{V(p',k)}{1 - R(p') + i\kappa(p')V(p',p')} \ , \tag{12}$$

with

$$R(p') \equiv m\mathbf{P}\int_0^\infty dk' \ \frac{k'^2 V(k',k')}{p'^2 - k'^2} \ . \tag{13}$$

Note that for an arbitrary function $g(k)$, such as $g(k) \equiv 1$ as discussed below, $R(p')$ may become a divergent integral. Also, if the potential is derived from field theory in general, the principal value integral turns out to be divergent. In these cases R is to be understood as properly regularized. The principal value integral $R(p')$ given above is therefore not only a model-dependent quantity, but also depends on the regularization scheme used. The condition that the on–shell NN scattering amplitude should satisfy eq. 5 relates this model– and regularization–dependent quantity to the on–shell potential, $V(p',p')$,

$$R(p') = 1 + \kappa(p')\cot(\delta(p'))V(p',p') \ , \tag{14}$$

where it is assumed that $\eta(p') = 1$. This shows that, for a given potential, the regularization should be such that eq. 14 be satisfied in order to reproduce eq. 5. Indeed, in conventional calculations based on meson–exchange models, where one introduces form factors to regularize the principal value integral, the cutoff parameters in these form factors are adjusted to reproduce the NN scattering phase shifts through eq. 5. Conversely, for a given regularization scheme, the NN potential should be adjusted such as to obey eq. 14. This is the procedure used in effective field theories [16], where the coupling constants in the NN potential become regularization–dependent.

We also assume that the production amplitude M is given by a separable form

$$M(E,k) = \beta g(k) h(p) \ , \tag{15}$$

where β is the coupling constant and $h(p)$ an arbitrary function of the relative momentum p of the two nucleons in the initial state. With the NN T–matrix and production amplitude given above the function $\mathcal{P}(E,p')$ given by eq. 3 can be written as

$$\mathcal{P}(E,p') = ap'\left(\frac{R(p')}{R(p') - 1}\right)\cot(\delta(p')) \ . \tag{16}$$

Thus, since $R(p')$ is a regularization–dependent quantity, so is the function $\mathcal{P}(E,p')$, as it was argued in the main section. Inserting this into eq. 7, and using eq. 14, we can express the total transition amplitude as

$$A(E,p') = -\frac{1}{\kappa(p')} e^{i\delta(p')} \sin(\delta(p')) \left(\frac{M(E,p')}{V(p',p')} \right) \ . \qquad (17)$$

Eq. 17 is the desired formula for our discussion. It allows us to study the relationship between the NN potential and the production amplitude $M(E,p')$ explicitly as different regularization schemes are used. For this purpose let us study the simplest case of a contact NN potential (setting the function $g = 1$ in eq. 11) in the limit of $p' \to 0$. If we regularize the integrals by means of the power divergent subtraction (PDS) scheme [16] we get

$$R = -\frac{a\mu}{1 - a\mu} \ ,$$

where μ denotes the regularization scale. Substituting this result into eq. 14, we obtain

$$\alpha = \left(\frac{2a}{\pi m} \right) \frac{1}{1 - a\mu}$$

for the NN coupling strength. Note that for $\mu = 0$ the PDS scheme reduces to that of minimal subtraction [16]. Since the total production amplitude A should not depend on the regularization scale we immediately read off from eq. 17 that

$$\beta \propto (1 - a\mu)^{-1} \ .$$

Therefore the model clearly exhibits the point made in the main section, namely, the requirement to calculate both the production amplitude and the FSI consistently in order to make quantitative predictions.

Eq. 16 allows to calculate explicitly the function $\mathcal{P}(E,p')$. We have

$$\mathcal{P} = -a\mu \ .$$

The minimal subtraction scheme, therefore, leads to $\mathcal{P} = 0$. The authors of ref. [16] argue, however, that $\mu \simeq m_\pi$ in order to provide a proper counting scheme for the NN interaction. In this case, however, we have $\mathcal{P} \simeq 6$ (here we used the pp scattering length of $a = -7.8$ fm).

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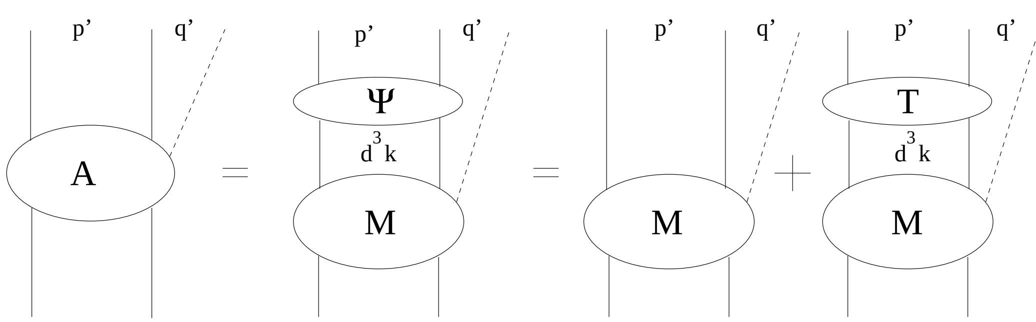

Fig. 1. *Decomposition of the production amplitude in the final state NN interaction and the production part. Here* Ψ *denotes the nucleon–nucleon wave function and* T *stands for the* NN T*–matrix.*